QCD and Spin Physics

Lecture 3

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DESY

The parton model

parton model description for DIS, Drell-Yan, etc.

- fast-moving hadron
  $\approx$ set of free partons with low transv. momenta
- physical cross section
  $=$ cross section for partonic process $(\gamma^* q \to q,\, q\bar{q} \to \gamma^*)$
  $\times$ parton densities

task:

- implement the parton-model ideas in QCD
  and correct them where necessary
  - identify conditions and limitations of validity
    (kinematics, processes, observables)
  - corrections: partons interact
    α_s small at large scales $\rightsquigarrow$ perturbation theory
  - definition of parton densities in QCD
    derive their general properties
    make contact with non-perturbative methods

Light-cone coordinates

$\rightsquigarrow$ blackboard

Kinematics of DIS

$\rightsquigarrow$ blackboard

Factorization from Feynman graphs

take inclusive DIS as example

- consider Bjorken limit, choose frame where
  - $p^+ \gg p^-$ (proton fast right-moving)
  - $q^+ \sim q^- \sim p^+$
  - $\boldsymbol{p}_T = \boldsymbol{q}_T = 0$
- for power counting
  - large: $p^+ \sim q^+ \sim q^- \sim Q$
  - small: hadron masses, scales of non-non-perturbative interact. $\sim m$
  - very small: $p^- \sim m^2/Q$

small expansion parameter is m/Q

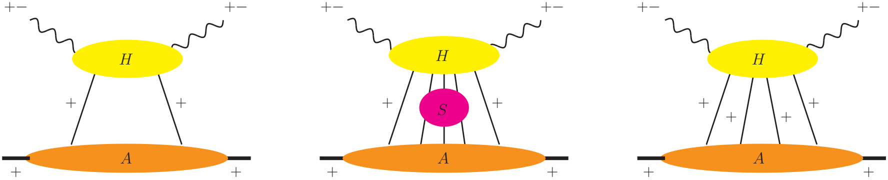
for power counting

- large: $p^+ \sim q^+ \sim q^- \sim Q$
- small: hadron masses etc. $\sim m$
- very small: $p^- \sim m^2/Q$

- in Bj limit graphs for $\gamma^* p \to \gamma^* p$ dominated by distinct momentum regions:
  - hard: $k^+ \sim k^- \sim k_T \sim Q$, $k^2 \sim Q^2$
  - collinear (to proton): $k^+ \sim Q$, $k_T \sim m$, $k^- \sim m^2/Q$, $k^2 \sim m^2$
  - soft: $k^+, k^-, k_T \ll Q$, $k^2 \ll Q^2$

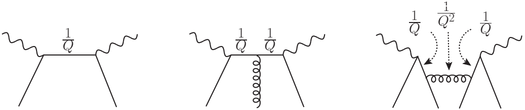
  proof involves advanced quantum field theory methods

- organize graphs into hard, collinear, and soft subgraphs



- power counting
  - hard subgraph $\propto Q^{\dim(H)}$
  - collinear subgraph $\propto m^{\dim(A)}$ (complications → later)
  - collinear lines:
    $d^4k = dk^+ \, dk^- \, d^2k_T \sim Q \times m^2/Q \times m^2 = m^4$
  - soft subgraph and lines: depends on detailed size of k^μ
- leading term: smallest possible number of lines to H

(complications → later)



tree-level hard graphs: no large k_T, but $k^+ \sim k^- \sim Q$

in loops: $k_T \sim Q$

Collinear expansion

- in hard graphs neglect small components of external coll. lines
  $\rightsquigarrow$ Taylor expansion

$$H(k^+,k^-,k_T) = H(k^+,0,0) + k_T^\mu \left[\frac{\partial H(k^+,0,k_T)}{\partial k_T^\mu}\right]_{k_T=0} + \mathcal{O}(m^2)$$

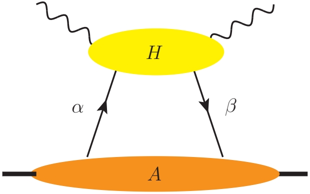
  first term $\rightarrow$ leading twist, second term $\rightarrow$ twist three, ...

- loop integration simplifies:

$$\int d^4k\, H(k) A(k) \approx \int dk^+\, H(k^+,0,0) \int dk^- d^2k_T\, A(k^+,k^-,k_T)$$

- in hard scattering treat incoming/outgoing partons as exactly collinear $(k_T = 0)$ and on-shell $(k^- = 0)$
- in coll. matrix element integrate over k_T and virtuality
  $\rightsquigarrow$ collinear (or k_T integrated) parton densities
  only depend on k^+

Complication from spin of partons (here: quarks, similar for gluons)



- H and A carry spinor indices:

$$H_{\beta\alpha} A_{\alpha\beta} = \mathrm{tr}(HA)$$

- use Fierz transformation
  $\rightsquigarrow$ $\mathrm{tr}(\gamma_\mu H)\ \mathrm{tr}(\gamma^\mu A)$ etc.

  - Lorentz invariance: in proton rest frame all components of
    $\mathrm{tr}\big(\gamma^\mu A(k,p,s)\big),\ \mathrm{tr}\big(\gamma^\mu \gamma_5 A(k,p,s)\big),\ \mathrm{tr}\big(\sigma^{\mu\nu}\gamma_5 A(k,p,s)\big), \ldots$
    are $\sim m^{\dim(A)}$ since $k^\mu, p^\mu, m\, s^\mu \sim m$
  - boost to Breit frame $\rightsquigarrow$ largest components
    $\mathrm{tr}\big(\gamma^+ A(k,p,s)\big), \mathrm{tr}\big(\gamma^+ \gamma_5 A(k,p,s)\big), \mathrm{tr}\big(\sigma^{+j}\gamma_5 A(k,p,s)\big)$
    are $\sim Q m^{\dim(A)-1}$ $\qquad j = 1,2$ transverse index
  - in Breit frame all components of $\mathrm{tr}\big(\gamma_\mu H\big), \mathrm{tr}\big(\gamma_\mu \gamma_5 H\big), \ldots$
    are $\sim Q^{\dim(B)}$

- up to power corrections have

$$\mathrm{tr}(HA) = \tfrac{1}{4}\Big[\mathrm{tr}(\gamma^- H)\,\mathrm{tr}(\gamma^+ A) + \mathrm{tr}(\gamma_5\gamma^- H)\,\mathrm{tr}(\gamma^+\gamma_5 A) + \tfrac{1}{2}\,\mathrm{tr}(i\sigma^{-j}\gamma_5 H)\,\mathrm{tr}(i\sigma^{+j}\gamma_5 A)\Big]$$

- coll. approx.: in H replace $k \to \bar{k}$
  with $\bar{k}^+ = k^+$, $\bar{k}^- = 0$, $\bar{k}_T = 0$

$$\begin{aligned}
&\int d^4k\ \mathrm{tr}(HA) \\
&\quad = \int dk^+\,\frac{1}{4}\,\mathrm{tr}\Big[\gamma^- H(\bar{k})\Big] \int dk^- d^2k_T\ \mathrm{tr}\Big[\gamma^+ A(k)\Big] + \{\text{other terms}\} \\
&\quad = \int \frac{dk^+}{k^+}\,\frac{1}{2}\,\mathrm{tr}\Big[\bar{k}^+\gamma^- H(\bar{k})\Big] \times \frac{1}{2}\int dk^- d^2k_T\ \mathrm{tr}\Big[\gamma^+ A(k)\Big] \\
&\qquad + \{\text{other terms}\}
\end{aligned}$$

- closer look at the factors $\rightsquigarrow$ blackboard

Definition of quark densities

- comparing with parton model result identify

$$f_1(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi}\, e^{ixp^+z^-} \left\langle p \middle| \bar{q}(0) \gamma^+ W(0, z^-)\, q(z^-) \middle| p \right\rangle$$

$$g_1(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi}\, e^{ixp^+z^-} \left\langle p \middle| \bar{q}(0) \gamma^+ \gamma_5 W(0, z^-)\, q(z^-) \middle| p \right\rangle$$

with parton densities:

$$f_1(x) = \begin{cases} q(x) & \text{for } x > 0 \\ -\bar{q}(x) & \text{for } x < 0 \end{cases}$$

$-$ from $dd^\dagger = -d^\dagger d$

$$g_1(x) = \begin{cases} \Delta q(x) & \text{for } x > 0 \\ \Delta\bar{q}(x) & \text{for } x < 0 \end{cases}$$

extra $-$ from helicity $= -$ chirality

- transversity $h_1(x)$ with $\gamma^+ \to i\sigma^{+j}\gamma_5$

Gluon densities

$$q(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi}\, e^{ixp^+z^-} \left\langle p \middle| \bar{q}(0)\gamma^+ W(0,z^-)\, q(z^-) \middle| p \right\rangle$$

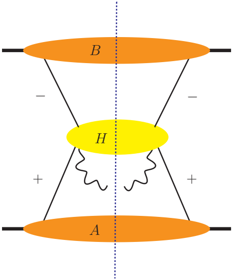
- for gluons replace

$$q(x) \to x g(x) \qquad\qquad \Delta q(x) \to x \Delta g(x)$$

$$\tfrac{1}{2}\bar{q}\gamma^+ q \to F^{+i} F_i{}^{+} \qquad\qquad \tfrac{1}{2}\bar{q}\gamma^+\gamma_5 q \to F^{+i} \widetilde{F}_i{}^{+}$$

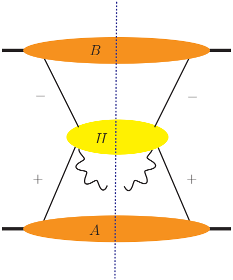
  with dual field strength $\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$
- understand extra factors x
  - $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
  - in light-cone gauge $A^+ = 0$ have $F^{+i} = \partial^+ A^i$
  - compare $\frac{1}{2}\bar{q}\gamma^+ q \to k^+$ with $F^{+i} F_i{}^{+} = (\partial^+ A^i)^2 \to (k^+)^2$

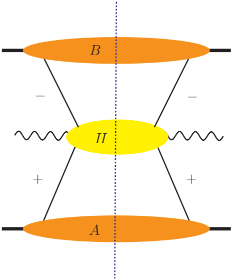
From DIS to Drell-Yan



- two collinear subgraphs for right- and for left-moving particles
- collinear factorization if
  - integrate over q_T of photon or
  - take $q_T \gg m$ large ($q_T \sim m$ in Friday lecture)

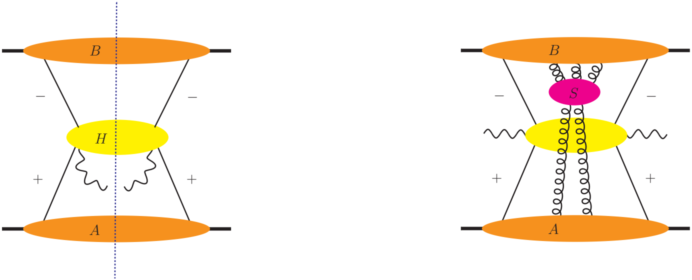
From DIS to Drell-Yan





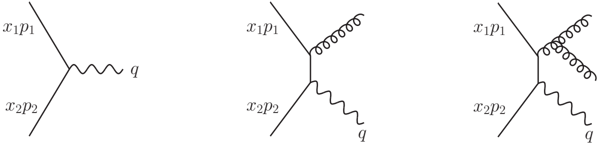
- two collinear subgraphs for right- and for left-moving particles
- no direct use of opt. theorem since γ^* fixed in final state
  trick: use crossing symm. to relate $pp \to \gamma^* X$ to $pp\gamma^* \to X$

From DIS to Drell-Yan



- two collinear subgraphs for right- and for left-moving particles
- no direct use of opt. theorem since γ^* fixed in final state
  trick: use crossing symm. to relate $pp \to \gamma^* X$ to $pp\gamma^* \to X$
- soft interactions between right- and left- moving spectators
  power suppr. only if sum over details of hadronic final state

Process vs. parton kinematics



- for $\int d^2 q_T$
  tree level $= \mathcal{O}(\alpha_s^0)$
  have $q_T \approx 0$
- measure $Q^2 \approx 2q^+ q^-$
  and $y = \frac{1}{2} \log \frac{q^+}{q^-}$
  $\rightsquigarrow \;\; q^+ = x_1 p_1^+, \;\; q^- = x_2 p_2^-$

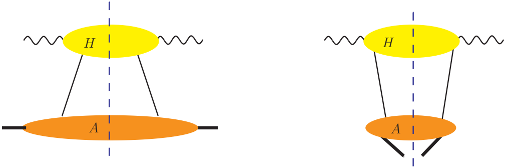
- for large fixed q_T
  tree level $= \mathcal{O}(\alpha_s)$
- measure $q_T, \;\; Q^2 = 2q^+ q^- - \boldsymbol{q}_T^2$
  and $y_q = \frac{1}{2} \log \frac{q^+}{q^-}$
  $\rightsquigarrow \;\; q^+$ and q^-
- $2(x_1 p_1^+ - q^+)(x_2 p_2^- - q^-) - \boldsymbol{q}_T^2 = 0$
- to fix x_1 and x_2 also need
  $y_{\text{jet}} = \frac{1}{2} \log \frac{x_1 p_1^+ - q^+}{x_2 p_2^- - q^-}$

More complicated final states

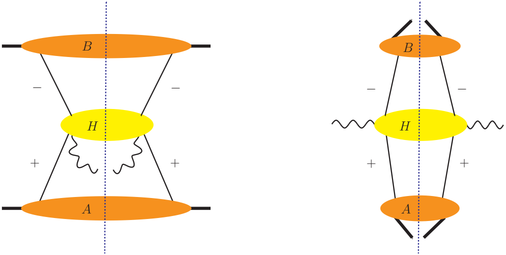
- production of W, Z or other colorless particle (Higgs, etc) same treatment as Drell-Yan
- jet production in ep or pp: hard scale provided by p_T
- heavy quark production: hard scale is m_c, m_b, m_t

Fragmentation

- cross DIS $eh \to e + X$ to $e^+e^- \to \bar{h} + X$
  i.e., $\gamma^* h \to X$ to $\gamma^* \to \bar{h} + X$



- or Drell-Yan $h_1 h_2 \to \gamma^* + X$ to $\gamma^* \to \bar{h}_1 \bar{h}_2 + X$



Fragmentation functions

- replace parton density $k^+ = xp^+$

$$f(x) = \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x}\, \langle h|\bar{q}(0)\Gamma^+ W(0,\xi^-)\, q(\xi^-)|h\rangle$$

$$= \sum_X \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x}$$

$$\times \sum_X \big\langle h \big| (\bar{q}(0)\Gamma^+)_\alpha W(0,\infty) \big| X \big\rangle \, \big\langle X \big| W(\infty,\xi^-)\, q_\alpha(\xi^-) \big| h \big\rangle$$

by fragmentation function $p^+ = zk^+$

$$D(z) = \frac{1}{2N_c z} \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+/z}$$

$$\times \sum_X \big\langle 0 \big| W(\infty,\xi^-)\, q_\alpha(\xi^-) \big| \bar{h} X \big\rangle \, \big\langle \bar{h} X \big\rangle \big| (\bar{q}(0)\Gamma^+)_\alpha W(0,\infty) \big| 0 \big\rangle$$

Summary of lecture 3

Factorization

- implements ideas of parton model in QCD
  - inclusion of perturbative corrections (NLO, NNLO, ...)
  - field theoretical def. of parton densities and fragmentation fcts.
    $\rightsquigarrow$ bridge to non-perturbative QCD
- valid for specified observables in specified kinematics
  - important results from general principles (power counting, ...)
  - soft spectator interactions complicate analysis
    for > 1 observed hadron (SIDIS, hadron-hadron coll., ...)
  - factorization proofs rather rare
- is an approximation scheme for large scales (Q, p_T, ...)
  - certain asymmetries zero in large-scale limit, progress in calculating $\frac{1}{Q}$ suppressed ($=$ twist three) observables
  - higher power corrections ($\frac{1}{Q^2}$ etc.) in general not calculable